GEARS: STRATEGIES IN PROBLEM SOLVING

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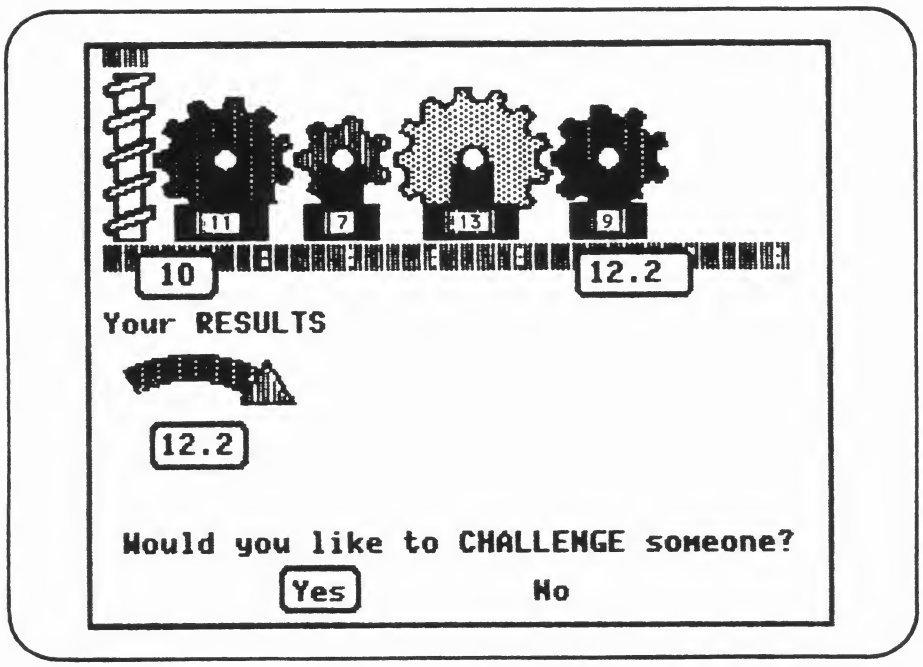
GEARS: STRATEGIES IN PROBLEM SOLVING

Program Objectives



Skills Needed: Multiplication and Division of Whole Numbers
Division of Decimals
Rounding of Decimals

Grade Level: 6th grade - adult

Reading Level: 6 (Fry)

Time Required: 10 - 20 minutes (per level)

Objectives:

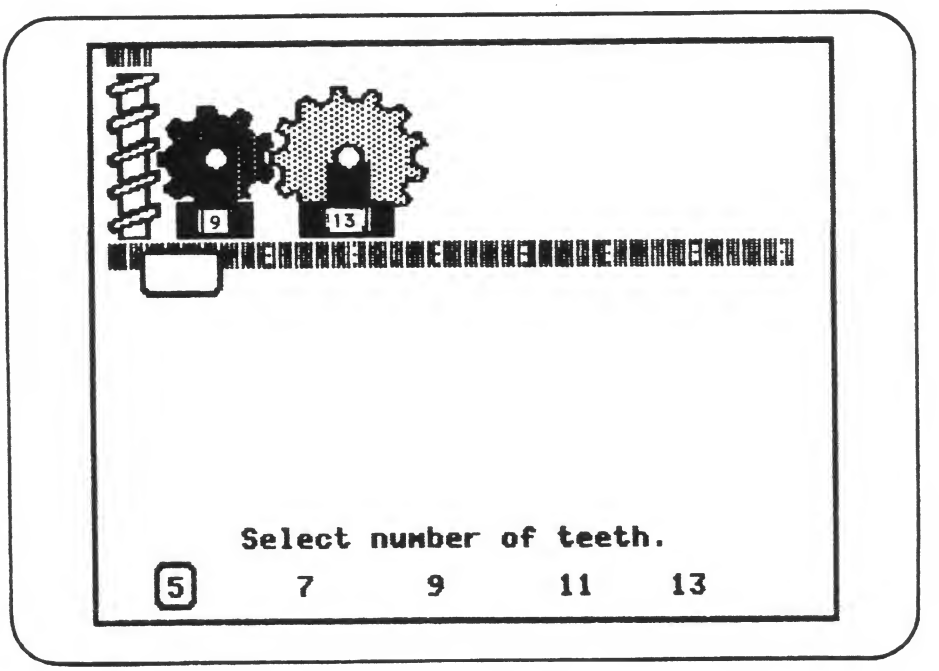
1. To put students in situations where they have to analyze a problem to identify a solution.
2. To allow students to discover mathematical formulas or build a model which will help them predict an outcome.
3. To increase skills involving division of decimal numbers.

GEARS: STRATEGIES IN PROBLEM SOLVING

Program Description

Overview

This program allows students to build valuable problem-solving skills as they experiment with the properties of gears. It contains three playing levels: Build a Gear Train, Solve a Gear Puzzle, and Build a Gear Factory. Students start by placing up to 4 gears (5 on the IBM and Tandy 1000) of various sizes (5, 7, 9, 11, and 13-tooth) together.



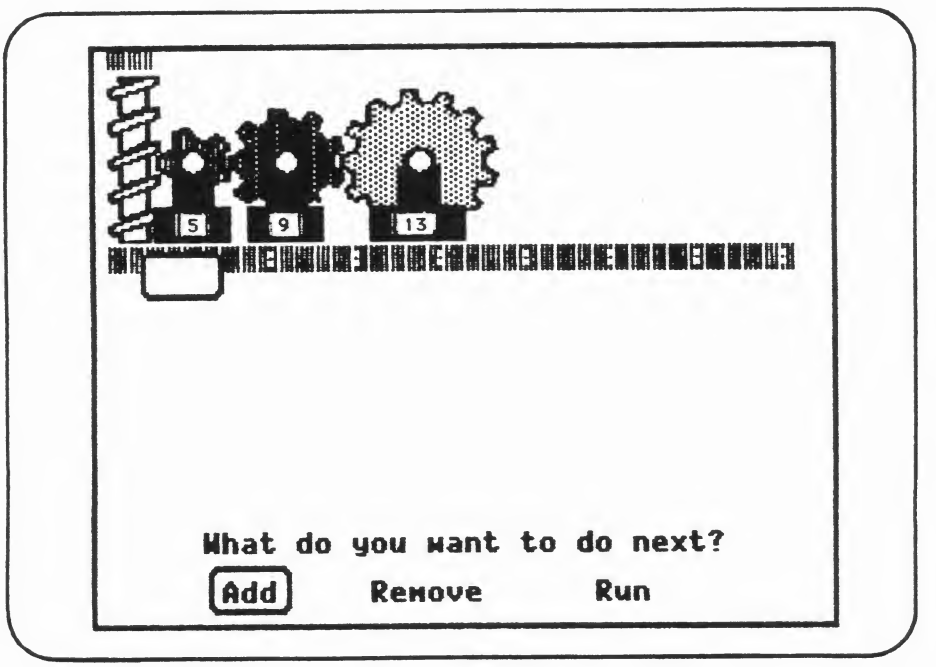
The motorized worm gear on the left of the screen drives all the gears that are connected to it. It always turns the first gear on its right in a counter-clockwise direction. That gear drives the next, which in turn drives the following gear, and so on. Students must discover the relationships or formula(s) that allow them to predict the direction and number of times the last gear will rotate.

After students experiment with different gear combinations, they can go to higher levels where they predict the results of a computer-generated problem.

Level One: Build a Gear Train

On this level, students can combine 2 to 4 gears (5 on the IBM and Tandy 1000) of various sizes (5, 7, 9, 11, and 13-tooth) in different combinations to build gear trains.

In the Build a Gear Train level, students are offered instructions. The playing screen then appears where they will build their gear train. First, they select the number of teeth for the first gear. That gear is then put into position on a gear platform. Students then have the option of removing that gear or adding another. All selections are made by using the arrow keys to move the indicator box and then pressing RETURN (Apple), ↵ (IBM PC/PCjr), or ENTER (TRS-80 Color).



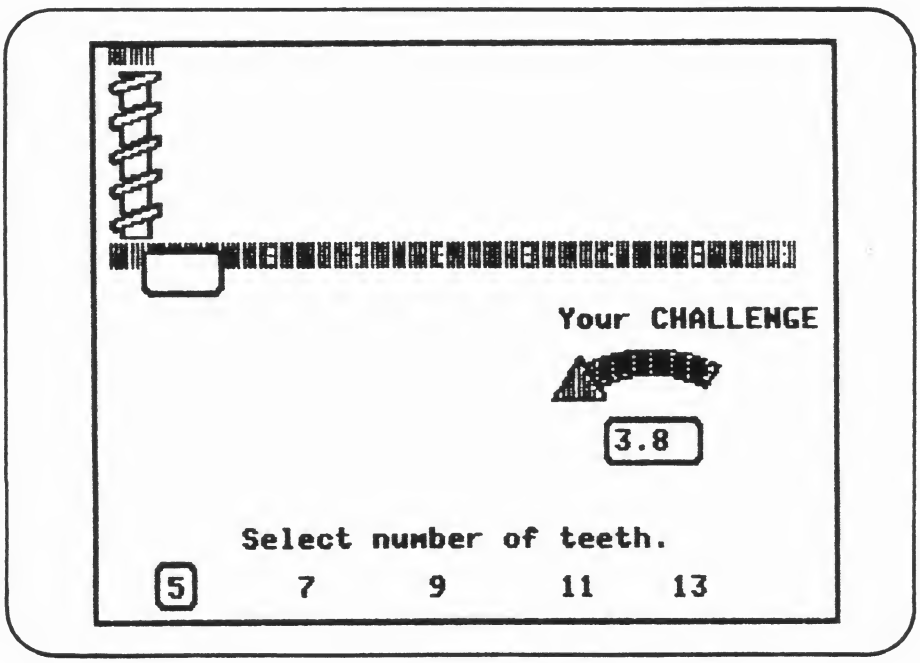
The first gear will always turn counter-clockwise for ten revolutions. Students should conduct a series of experiments, taking careful notes to determine the number of turns for the last gear and the rotational direction.

For example, if students build a train with a 5-tooth, a 9-tooth, and a 13-tooth gear, the first will always turn counter-clockwise exactly 10 times, while the last will turn counter-clockwise 3.8 times. If students then build another train with only a 5-tooth and 9-tooth gear, they will find that the 9-tooth gear turns clockwise for 5.6 revolutions. If they then build a train with a 9-tooth, a 13-tooth, and two 5-tooth gears, they will discover that the last (5-tooth) gear turns clockwise for exactly 18 revolutions.

Throughout the program, students have the option of speeding up the gear process by pressing the ESC key (or the CLEAR key on the TRS-80 Color). This will interrupt a RUN and display the results.

Analysis of the models is important to the discovery of the mathematical formulas used in every gear train. The Facts About Gear Trains on page 13 explains this in detail.

The two number boxes under the gear train indicate the number of revolutions of both the first and the last gear. Students can record the result and run another experiment, or can choose to challenge other students to duplicate their result.



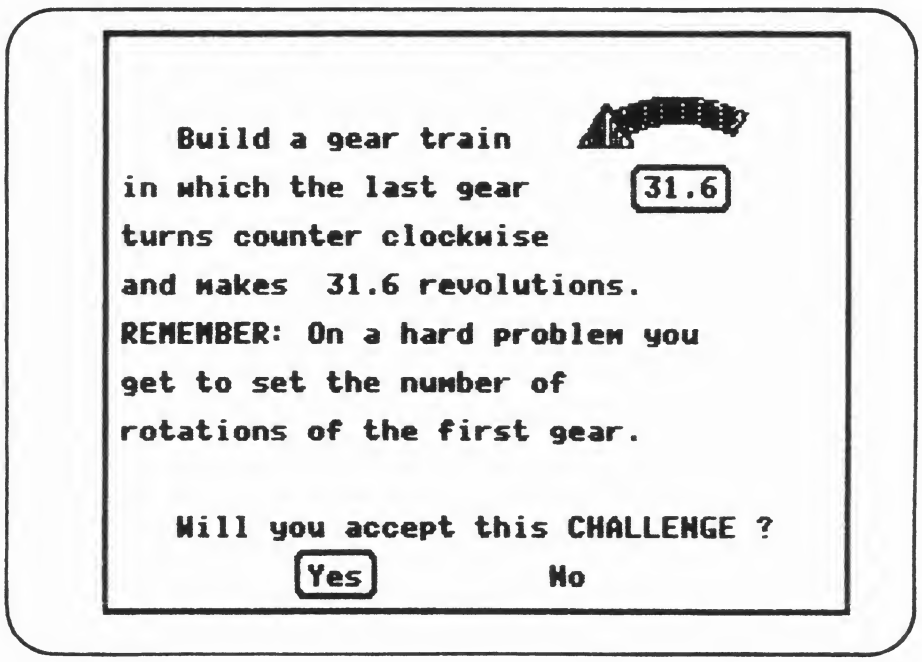
The challenge screen presents the direction and the number of revolutions previously reached by the final gear. This is the challenge the next student must try to duplicate. The number or types of gears need not be the same, only the result must be the same.

The students should experiment in this manner until they can predict the cause-effect relationships of the gear train. At that point they can advance to Levels Two and Three, where the new learned knowledge is tested.

Level Two: Solve a Gear Puzzle

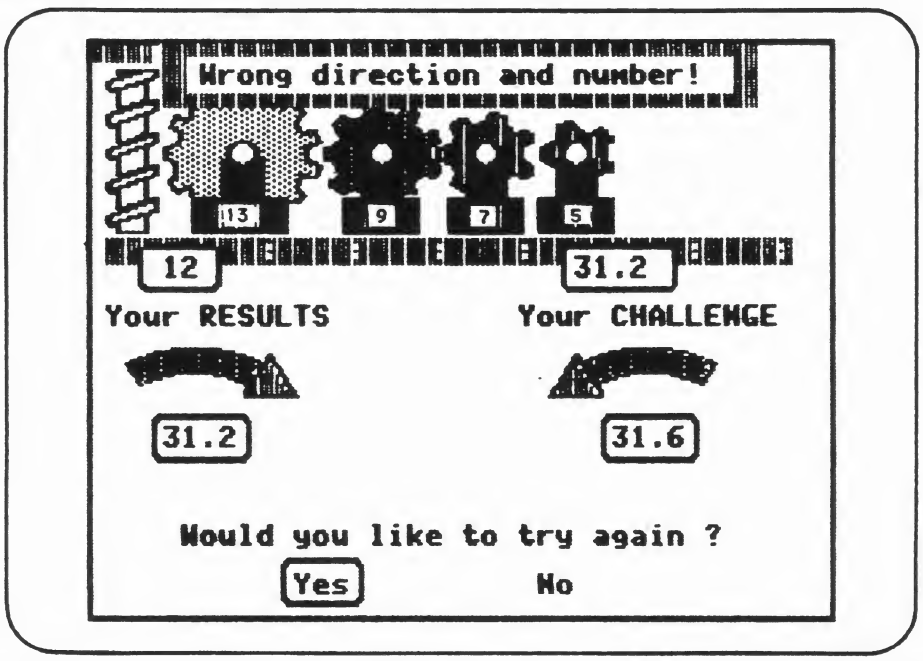
This level requires that students solve a gear puzzle determined by the computer. The computer tells the student how many times the last gear in the train must turn, and the student must then build a train that produces the desired result.

As on Level One, students are first offered instructions. They are then asked to select an "EASY" or a "HARD" problem. If easy problems are selected, the first gear always rotates 10 times counter-clockwise. If hard problems are chosen, the students get to determine the number of rotations of the first gear again in a counter-clockwise direction. After choosing easy or hard, the computer's challenge is displayed as shown below:

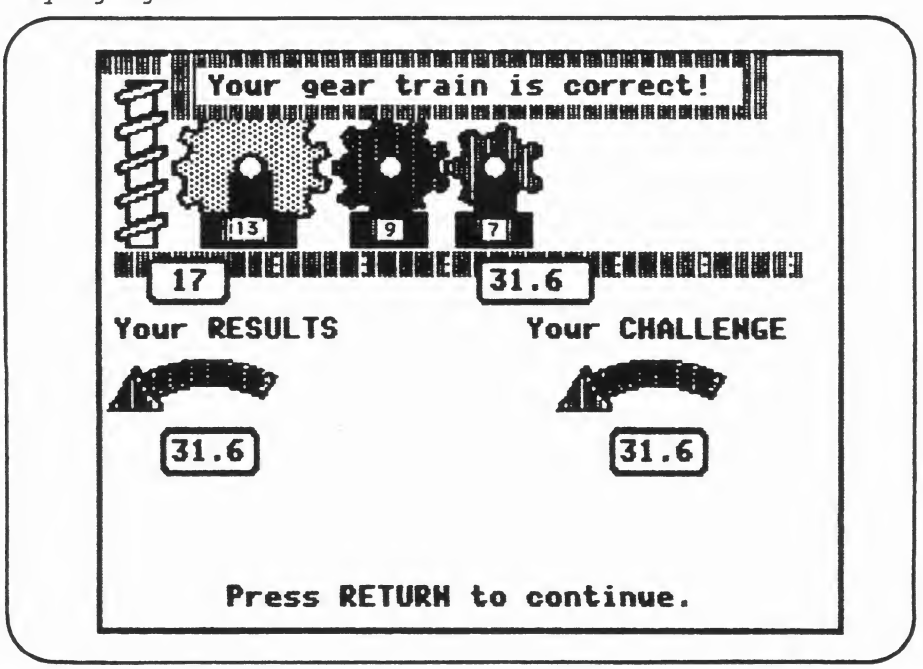


Students can choose whether or not they accept the challenge. If the students answer "No," they are given the choice of getting another challenge or returning to the main menu. If the students choose to see another challenge, they may again choose an easy or a hard problem. After selecting to accept a challenge, students are then taken to the playing screen where they will build the gear train. Gears are again designed and put together by using the arrow keys and RETURN key to select their size and number. After at least two gears are put together, the student may choose to run the train. Students who have chosen a hard problem may at that point use the right- and left-arrow keys to select the number of complete revolutions for the first gear.

As the program is run, the computer displays the number of revolutions for the first and last gear in the train, along with the challenge number that the students are trying to attain.



The gear train runs and the students observe the results. If the gear train does not produce the desired number of revolutions, students are given the option of trying again.



If they have correctly solved the gear puzzle, they are congratulated and then may try another or move to a new level.

Level Three: Build a Gear Factory

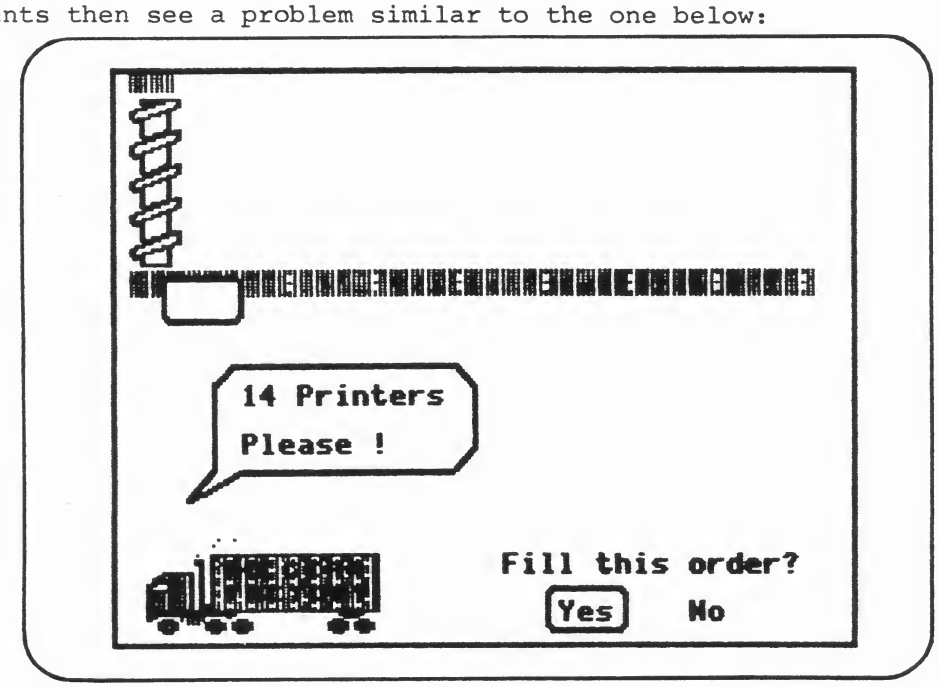
On this level, students must build a "gear factory" that will fill orders determined by the computer. They again use 2 to 4 gears (5 on the IBM and Tandy 1000) with teeth of 5, 7, 9, 11, or 13. This level offers instructions followed by the choice of either an "EASY" or a "HARD" problem.

If the students choose an easy problem, the first gear always rotates 10 times. Students will be presented with a certain number of products to produce. One product is produced with each complete rotation of the last gear. Therefore, 14.8 revolutions will produce 14 products since there were only 14 complete revolutions.

If the students choose the hard problem, they will also be presented with a number of products to produce but the students are allowed to choose the number of revolutions of the first gear (1 to 30). So that the students cannot merely choose the same number of rotations as the number of products needed, that exact number is not allowed as one of the choices possible for the first gear.

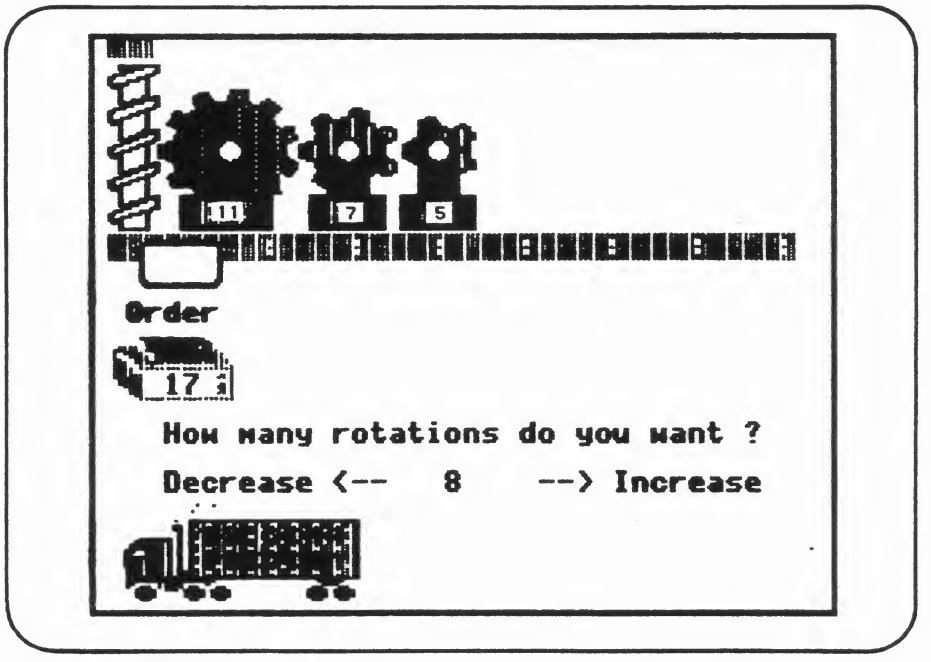
The direction that the last gear turns determines what type of product the factory makes. If the last gear turns clockwise, the factory makes monitors. If the last gear turns counter-clockwise, the factory makes printers.

The students then see a problem similar to the one below:

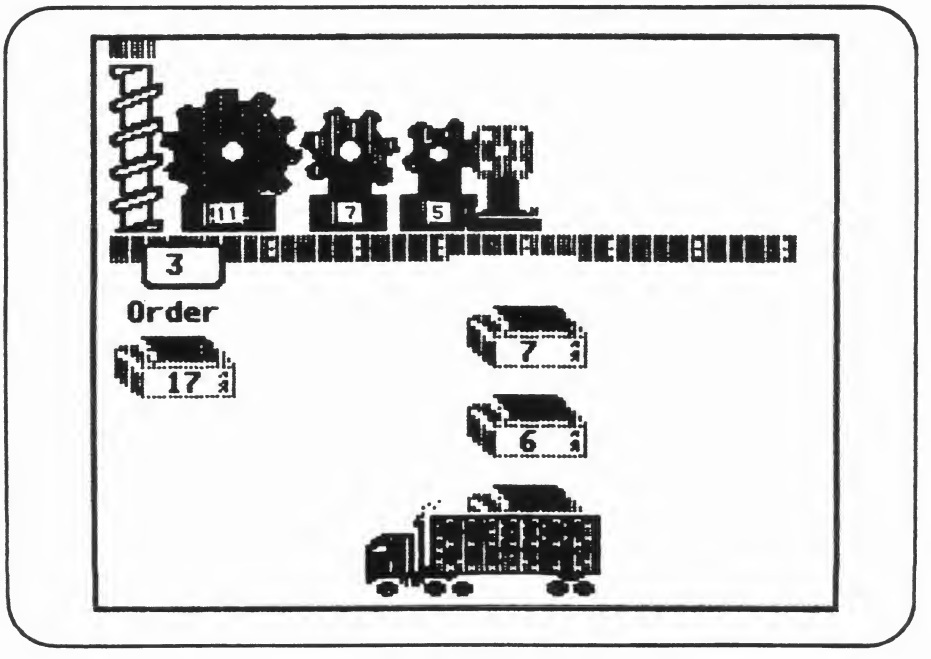


Students are asked if they wish to fill that order, get a new order, or return to the menu. If they choose to fill the order, they again choose the gears which will produce the correct number and types of products.

If students choose a hard problem, they now use the right- and left-arrow keys to select the number of complete revolutions for the first gear, as shown below:



As the gear train is run, each time the last gear makes one complete revolution, a product drops from the gear train into the waiting truck below. The number of products produced is indicated along with the number ordered. If the results do not fit the order, students are given the choice of trying again. If the results fit the order, they are congratulated.



Displaying the Result

Once students have become familiar with the program, they can speed up a game by pressing the ESC key (or the CLEAR key on the TRS-80 Color). This will interrupt a RUN and display the final result.

Turning Sound On or Off

On the Apple, a Sound Option is available allowing students to toggle between sound on/off. This can be done by holding down the CONTROL (CTRL) key and pressing S.

On the IBM PC/PCjr and Tandy 1000, a Change Option is available allowing the teacher to turn the sound off and on again. This can be done by holding down the Control (CTRL) key and pressing T. When using the sound option, you will first see a screen displaying the current status of the Change Option. To turn the sound on or off, answer "Yes" or "No" to the question: "Do you want sound?" The computer will then remain in either mode until the option is used again.

Once the sound has been turned on by the teacher, students can then toggle between sound on/off. This can be done by holding down the CTRL (CONTROL) key and pressing S (IBM) or A (Tandy 1000).

On the TRS-80 Color, the sound control is on the television.

Exiting the Program

Students may stop the program at any time by doing the following:

- Apple, IBM PC/PCjr, Tandy 1000: hold down the CONTROL (CTRL) key, and press the E key.
- TRS-80 Color: hold down the SHIFT and down-arrow keys, and press the E key.

GEARS: STRATEGIES IN PROBLEM SOLVING

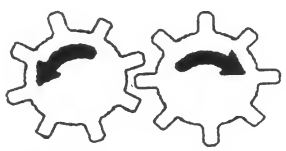
Facts About Gear Trains

This section is strictly for the teacher. Students should learn these concepts through using the program.

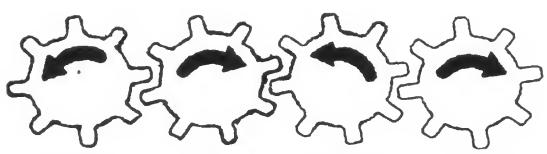
Rotational Direction

It is basic knowledge that a gear turning one direction will cause the gear it is meshed with to turn the opposite direction given the fact that the first gear in the program is always driven in a counter-clockwise direction, the following facts can be derived:

- Clockwise Rotation of Last Gear
  Students wishing to make the last gear turn in a clockwise direction should construct a train with either two or four gears.

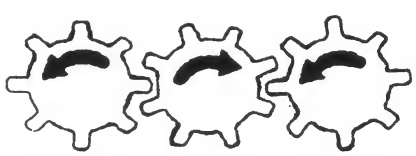


counter-clockwise clock-wise



counter-clockwise clock-wise counter-clockwise clock-wise

- Counter-Clockwise Rotation of Last Gear
  Students wishing to make the last gear turn in a counter-clockwise direction should construct a train with three gears.



counter-clockwise clock-wise counter-clockwise

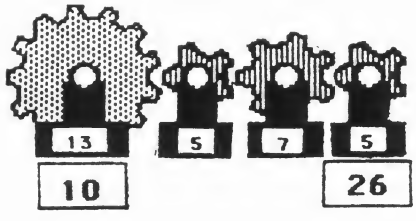
Number of Revolutions With Two Gears

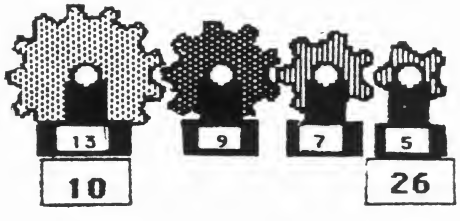
As students work with models of meshed gears, it is easy to see that if there are two gears, some basic facts can be derived:

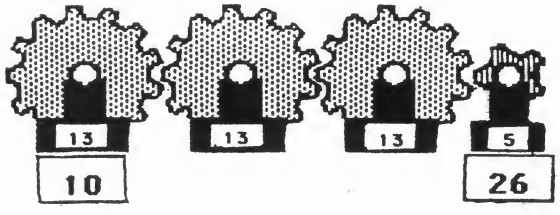
- If the two gears are the same size, then the number of complete revolutions of the first gear will equal the number of complete revolutions of the second gear.
- If the first gear is larger in size than the second gear, then one complete revolution of the larger gear will cause the smaller one to rotate more than one complete revolution. Therefore, in this case, the second gear will always rotate more times than the first gear.
- Conversely, if the first gear is smaller in size than the second gear, then one complete revolution of the smaller gear will not cause the larger gear to rotate to a complete revolution. Therefore the second gear will always rotate less times than the first gear.

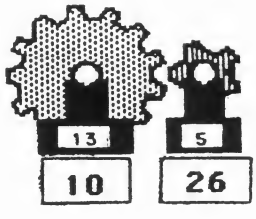
Number of Revolutions With More Than Two Gears

As students work with more than two gears, they will notice that the number of gears only affect the DIRECTION of the final gear. The number of rotations of the final gear only depends on the first and last gears. Students may place gears of different sizes between the first and last gear and the number of turns made by the final gear will not change.







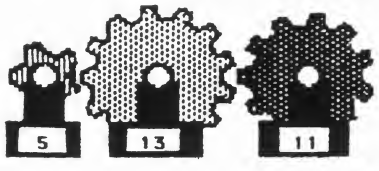


The Formula

Of course, there is a formula that can be used to determine the exact number of times the last gear in the train will rotate:

$$\text{Revolutions of last gear} = \frac{(\#\text{ of teeth of first gear}) \times (\text{revolutions})}{\#\text{ of teeth of last gear}}$$

For example, if you have the following gear train with the first gear rotating ten times:

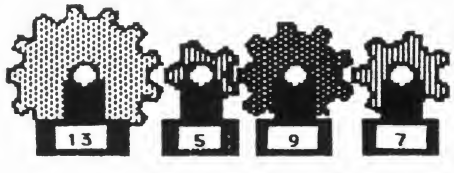


Using the formula above, you get:

(# of teeth of first gear) x (revolutions) = (5) x (10)
# of teeth of last gear = 11

$$\text{Revolutions of last gear} = \frac{5 \times 10}{11} = 4.5$$ rounded to the tenths place

Another example:



(# of teeth of first gear) x (revolutions) = (13) x (10)
# of teeth of last gear = 7

$$\text{Revolutions of last gear} = \frac{13 \times 10}{7} = 18.6$$ rounded to the tenths place

Classroom Use

Introducing Students to Gears

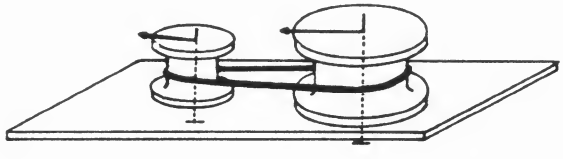
Begin introducing the program to your students by giving them a short overview of the history of gears:

> Gears first appeared over two thousand years ago. They were originally developed as a leverage device that allowed people to lift or push heavy loads. The most common use of early gears was to draw water from deep wells.
>
> Later, larger gears were made so that animals could draw up the water. As time went on, gears were used with windmills and water wheels to grind grain into flour.
>
> The worm-type gear, shown on page 5, was developed by Archimedes in Greece to drive other gears. Leonardo da Vinci was another famous historical figure who understood and made innovative use of the intriguing property of gears. Students could be given an assignment to write a report on one of these men.
>
> Today, gears are used in many every-day objects. Non-digital watches and clocks make extensive use of gears. Bicycles use gears; they can make your pedaling very easy when you go up a hill, or they can make you go very fast when you come back down. A car is full of gears. Even television dials work with gears.

Next, you may want to give a classroom demonstration of some common objects or tools that use gears. Some suggestions:

an egg beater	a can opener
a pencil sharpener	record player 33rpm/45rpm/78rpm
a bicycle	disect an old clock or watch
a toy train	

Students might also look at the movement of "pulleys." Two different sized spools of thread with rubber bands connecting them could be experimented with as shown below:



With a pointer marked on each spool, the students could discover the relationship of the rotations of the two pulleys. Then, as they begin to work with meshed gears, they will discover the basic difference between pulleys and meshed gears is a change of direction.

Classroom Use of Build a Gear Train

The easiest concept for beginners to understand is the direction of the rotation of meshed gears. Using the Build a Gear Train, the students should begin by working with as many combinations of gears as possible. The types of gears used are not important at first, but the rotation of the last gear should be noted along with the number of gears used. After experimenting with the program and predicting results for a short time, students should be able to state the relationship listed on page 13. Once they are comfortable with the direction of the rotation, hand out the Rotation Worksheet (page 23) as a follow-up.

Next, have the students discover the relationships gears of different sizes have on each other. Using the Gear Ratio Worksheet (page 24) will help students keep track of the revolutions in every two-meshed gear combination. Since most of the combinations have decimal answers and are derived from a formula using division, students should have worked with division of decimals and rounding to the tenths place prior to doing this section. GEARS is an excellent practical application for both concepts.

Students should fill in the Ratio Worksheet. As they work with more than two gears, they soon discover that the number of gears only changes the direction; the number of rotations is only dependent on the first and last gears. From their notes they should also begin to see the relationships between the number of rotations and the number of teeth.

As they become more familiar with how the gears work, they may use the challenge part of Build a Gear Train to challenge a friend to duplicate the number of rotations made by the last gear. At this point, if they haven't done so already, they should be told to look for a formula for the gears rotations. Many students may find the formula on their own, but others may need a little extra guidance.

Classroom Use of Solve a Gear Puzzle

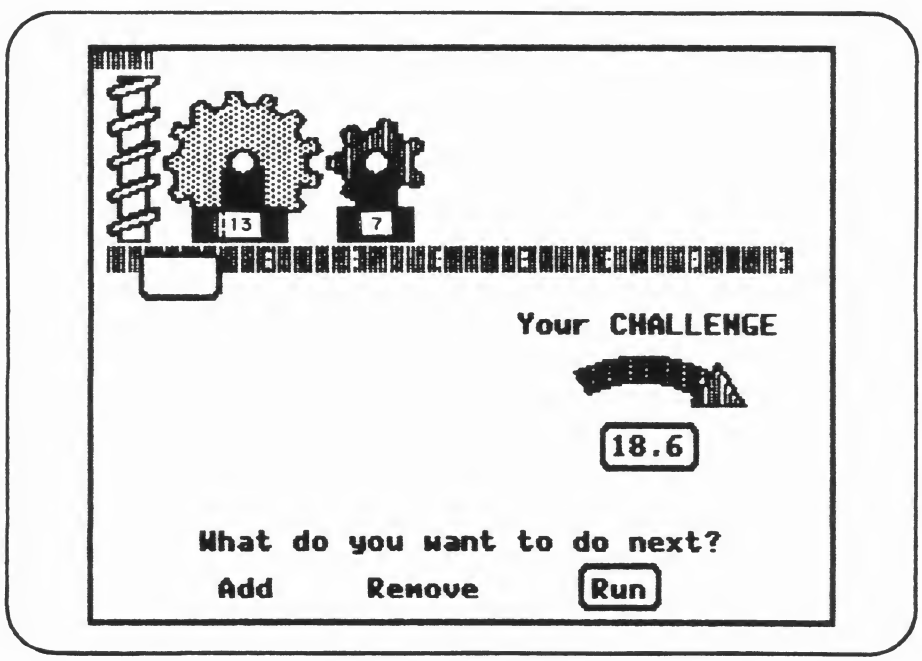
Even though it's helpful, it's not necessary for students to find the formula in order to proceed to the second level. Many students may need this level to motivate them. In Level Two there are easy and hard problems. On the easy problems, the number of revolutions of the first gear is always 10. On the hard problems, the students may, after putting the gears together, choose to have from 1 to 30 revolutions for the first gear. Students should stay on the easy level until they see the relationships and derive the formula.

As students begin to use the formula:

$$\text{Revolutions of last gear} = \frac{(\#\text{ of teeth of first gear}) \times (\text{revolutions})}{\#\text{ of teeth of last gear}}$$

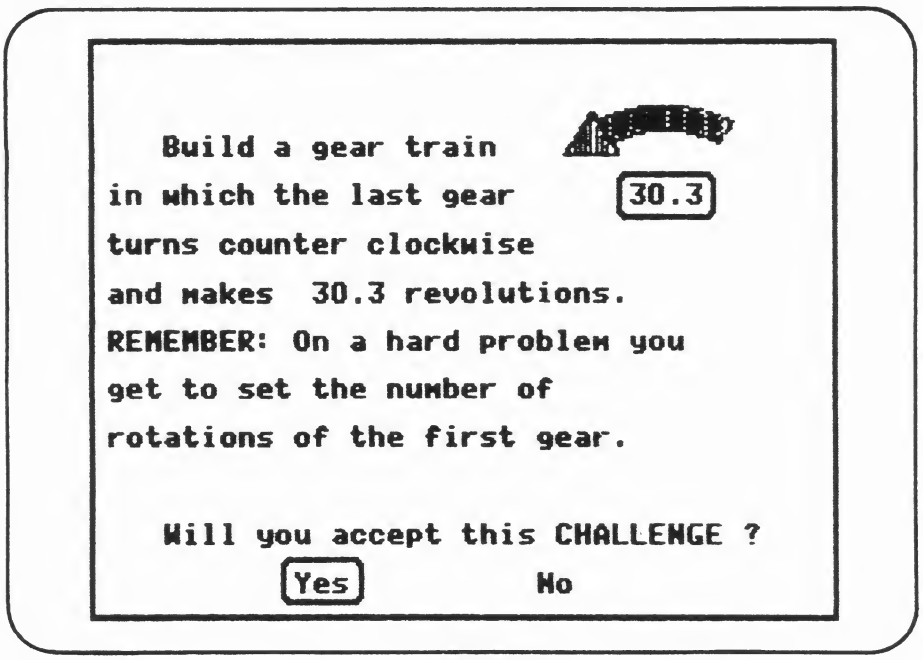
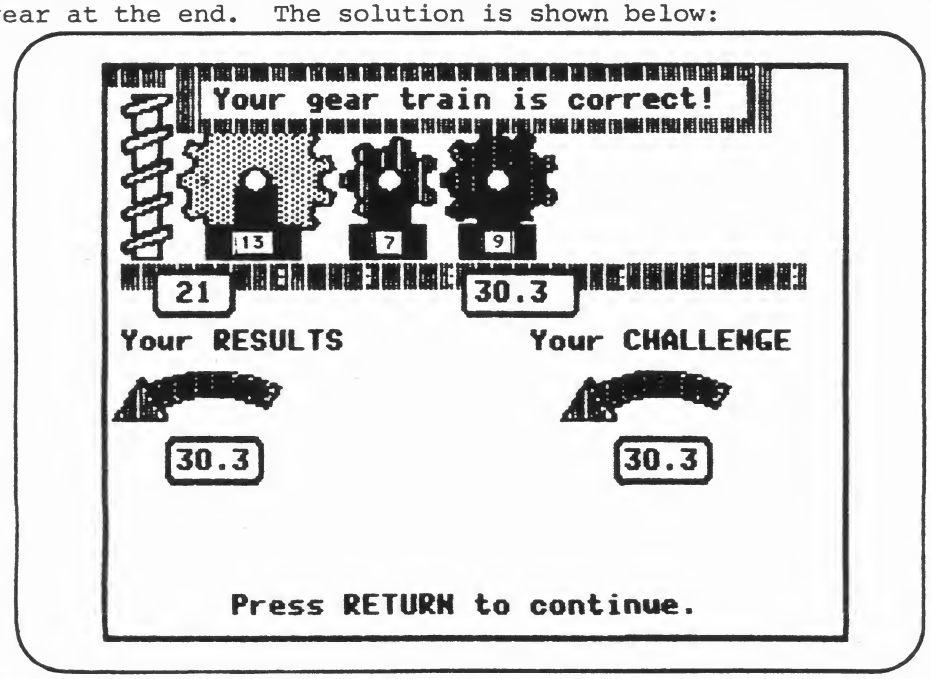
they should be reminded that this is a good place to practice their rounding skills. The number of revolutions is rounded to the tenths place.

An easy problem could be similar to the one below. Students know the first gear will rotate 10 times and must make the last gear rotate 18.6 times:



The solution could have had 2 or 4 gears to achieve the clockwise directions. Students could use the formula or the Ratio Worksheet to form their conclusions as to which gears to use.

A hard problem may not be possible with the first gear rotating 10 times. For example, on the screen below, the object is to make the final gear rotate 30.3 times.



This would be impossible to do as an easy problem since the maximum number of revolutions would be 13 x 10 divided by 5, or 26 revolutions. Since on hard problems the number of revolutions may be varied, one solution would be to use a 13-tooth gear first, rotate it 21 times, any gear in the middle and a 9-tooth gear at the end. The solution is shown below: